Reconstruction of Cross-Wind Profile from Turbulent Intensity Fluctuations of Diffusely Scattered Optical Wave

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INTRODUCTION

The problem of measuring the mean (averaged over path) wind velocity based on the spatio-temporal statistics of fluctuations of intensity of optical wave or light scattered off a viewed object surface or natural scene in a turbulent atmosphere was investigated in[1,2]. The results of wind profiling by SCIDAR technique are presented, in particular, in[3]. In this paper we consider the problem of wind profiling from the turbulent intensity fluctuations of reflected wave. We present the basic equations for reconstruction of wind velocity and direction profiles from the spatiotemporal spectrum of intensity fluctuations of optical wave scattered by a diffuse target in a turbulent atmosphere. We analyze the intensity of scattered wave being recorded in the focal plane of the receiving telescope. The results of end-to-end computer experiments on reconstruction of wind profiles we present as well.

FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

Let us consider the following scheme. A laser source in the plane $x' = 0$ illuminates a diffuse surface located in the plane $x' = x$ as is shown in Fig. 1. The reflected radiation in the plane of the source is passed through a telescope objective with focus length F_t and radius a_t, and recorded by an array of receivers (a video camera) placed at $x' = -l$. The speckle images of a laser illuminated area on a diffuse surface are subject of correlation-spectral processing.

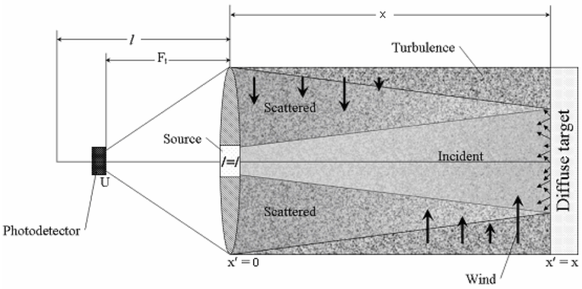


Figure 1. Geometry of the problem.

For simplicity, below we list the basic equations for the spherical wave incident on infinite diffusely scattering surface. Then for the spatiotemporal correlation function of intensity at the focal plane of receiving telescope

$$K_I(\rho_1, \rho_2; \tau) = \langle I(F_t, \rho_1, 0)\, I(F_t, \rho_2, \tau)\rangle - \langle I(F_t, \rho_1, 0)\rangle \langle I(F_t, \rho_2, \tau)\rangle, \qquad (1)$$

using the statistics of diffuse surface reflecting coefficient[4], the Taylor's hypothesis of frozen turbulence[5], the Kolmogorov's model of atmospheric turbulence[6], and assuming the regime of weak intensity fluctuations on propagation path[4], we obtain

$$K_I(\rho, \tau) = 4A \int d\boldsymbol{\kappa} \int_0^x dx' \Phi_\varepsilon(x', \boldsymbol{\kappa}) \times e^{i\boldsymbol{\kappa}\mathbf{V}(x')\tau - i\xi(x/F_t)\boldsymbol{\kappa}\rho - ((\xi^2 a_t^2)/2)\kappa^2} \sin^2\left(\frac{x\kappa^2}{2k}\xi(1-\xi)\right), \qquad (2)$$

where $I(F_t, \rho, \tau)$ is the wave intensity, and the angle brackets $\langle\ldots\rangle$ denote ensemble averaging, $\mathbf{V}(x')$ is the wind velocity component transverse to the propagation direction, τ is time, $\rho_1 - \rho_2 = \rho = \{y, z\}$, $k = 2\pi/\lambda$, λ is the wavelength, $\Phi_\varepsilon(x', \boldsymbol{\kappa})$ is the 3D spectrum of dielectric permittivity fluctuations[6], $\xi = x'/x$, A is a constant.

Let us denote $\mathbf{q} = q\mathbf{e}_i$; $\mathbf{V}(x')\mathbf{e}_i = V_i(x')$; $\alpha = \omega/q$. Then, based on the spatiotemporal spectrum

$$g(\alpha, q) = \frac{1}{(2\pi)^3}\int d\tau d\rho\, K_I(\rho, \tau)\, e^{i\mathbf{q}\rho - i\omega\tau}$$

after scaling $g_0(\alpha, q) = \dfrac{q^2 g(\alpha, q)}{4A\Phi_\varepsilon(\mathbf{q})} \exp\left(q^2 \dfrac{F_t^2 a_t^2}{2x^2}\right)$

and integration $G(\alpha) = \int_0^\infty g_0(\alpha, q)\cos(pq^2)\, dq$,

we obtain

$$G(\alpha) = \frac{\pi}{8}\int_0^x dx'\, C_n^2(x')\left(\frac{x\xi}{F_t}\right)^{5/3} \delta\left(\alpha - \frac{F_t V_i(x')}{x\xi}\right) \times \left[2\delta(p) - \delta\left(p + \frac{F_t^2}{2kx}\,\frac{1-\xi}{\xi}\right) - \delta\left(p - \frac{F_t^2}{2kx}\,\frac{1-\xi}{\xi}\right)\right], \qquad (3)$$

where C_n^2 is the structure characteristics of refractive index[6]. In the region $p > 0$, each value of ξ corresponds to a relevant δ-spike at $p = \frac{F_t^2}{2kx}\,\frac{1-\xi}{\xi}$. The velocity is determined by the relation $\alpha = \frac{F_t\,V_i(x')}{x\xi}$. Tracing to maxima of Eq. (3) we obtain profile $V_i(x')$ along a path between transmitter-receiver and diffuse surface.

CALCULATION ALGORITHM AND NUMERICAL RESULTS OF WIND RETRIEVAL

In case of infinite reflecting diffuse surface the intensity of the reflected wave at the focus of the receiving telescope can be written in the form[7]

$$I(F_t, \rho) = \int d\mathbf{r}\, I_I(x, \mathbf{r})\, I_T(x, \mathbf{r}, \rho)\,. \qquad (4)$$

In Eq. (4), the intensity $I(F_t, \rho)$ is the integral of the product of intensities of a beam illuminating the surface, $I_I(x, \mathbf{r})$, and the beam with the initial radius which equals the telescope radius, propagating to the diffuse target under the angle determined by the vector ρ and the focal length F_t, $I_T(x, \mathbf{r}, \rho)$. Hence, the algorithm for numerical simulation is following. Propagation of the illuminating and "telescope" beams in the forward direction is modeled, for example, by the split method[8], their intensities are calculated, multiplied together, and summed up over the whole illuminated area. But strict algorithm (4) requires too much computer time. That is why in computer simulation we use the approximation[7]:

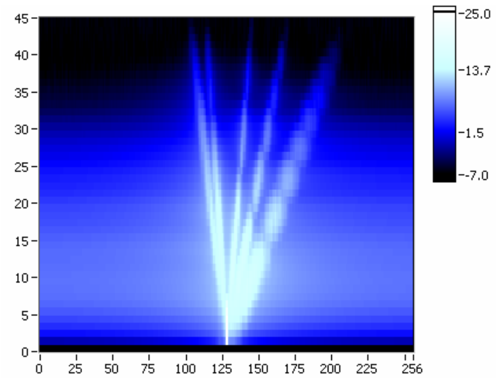
$$I(F_t, \rho) \simeq \int d\mathbf{r}\, I_I^2(x, \mathbf{r})\, I_T^0(x, \mathbf{r}, \rho), \qquad (5)$$

where $I_T^0(x, \mathbf{r}, \rho)$ is calculated for the "telescope" beam propagating in absence of turbulence.

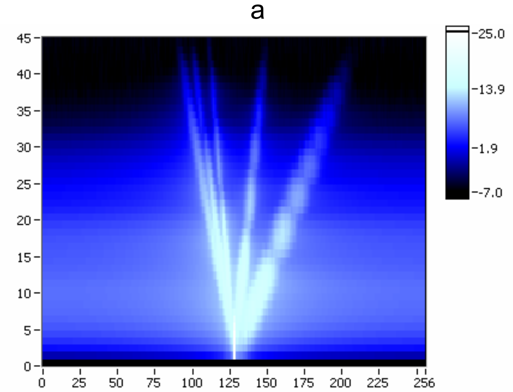
To calculate the spatiotemporal spectrum $g(\alpha, q)$, we have performed the computer simulation of the illuminating and telescope beam propagation in a turbulent atmosphere using the code[99] based on the splitting method[8] of numerical solution of the scalar parabolic equation. Turbulence was simulated through phase distortions of the propagating wave using equidistant random phase screens with the Kolmogorov spectrum[6] of phase inhomogeneities. The parameters of the screens were chosen so that the necessary requirements on modeling accuracy were met and the regime of weak optical turbulence was realized in simulation.

Wind velocity variations over the path were imitated by time-varying positions of the phase screens to the distance $\mathbf{V}(x')\cdot\tau$, where x' is the current longitudinal coordinate, in accordance with the specified wind profile. Simulated random 3-D intensity samples (2-D in space and 1-D in time) were then fast-Fourier transformed over time and space and summed over both coordinates with the weight filtering functions according to Eq. (3). Then, we built the wind profile by algorithms (3).

As an example of wind profile retrieval by the developed algorithm we present the results obtained for the following initial data: path length $x = 1000$ m, wave length $\lambda = 0.5$ µm, $C_n^2 = 4\cdot10^{-17}$ m$^{-2/3}$, $F_t = 1$ m, $a_t = 5$ cm, divergence of illuminating beam $\theta = 5\cdot10^{-4}$ radian. Logarithm of the spatio-temporal spectrum for the case of simulation of 5 moving screens is shown in Fig. 2.



a



b

Figure. 2. Spatio-temporal spectrum of intensity fluctuations. a – *z*-component, b – *y*-component. Spatial and temporal frequencies are plotted on vertical and horizontal axes, correspondingly.

The spectrum was estimated from 10 realizations of duration of 256 frames of 2-D intensity distributions of 512×512 pixels. Each phase screen is presented in the spectrum by a strip with the origin on the Nyquist frequency (128's point).

The results of wind retrieval by the developed algorithm are listed in Table 1. As it is seen from the Table, the wind profile is reconstructed with acceptable accuracy for 2/3 of the path adjacent to transceiver. The reason why the wind profile is not resolved on the one third part of path adjacent to the reflector is the use of approximation (5) in computer simulation instead of strict relation (4) rather than imperfection of the algorithm (3).

The numerical experiment can be significantly simplified as well, if we present the integral over **r** in Eq. (4) as the convolution integral. This is possible, if we consider the refractive index fluctuations located within a thin layer near the source. For the model of random phase screen located near the light source, Eq. (4) is represented as:

$$I(F_t, \rho) = \int d\mathbf{r}\, I_l(x, \mathbf{r})\, I_T(x, \mathbf{r}, \rho) =$$

$$= \int d\mathbf{r}\, I_l(x, \mathbf{r})\, I_T(x, \mathbf{r} - \rho)\,. \qquad (6)$$

The model of random phase screen is used frequently in the analysis of wave propagation along vertical and slant paths in a turbulent atmosphere.

Table 1. Initial and reconstructed wind profiles.

Initial						Reconstructed					
x, m	0.0	200	400	600	800	*x*, m	10	180	430	610	-
V_z, m/s	-3.0	-2.0	-1.0	1.0	2.0	V_z, m/s	-3.10	-1.9	-0.8	0.7	2.3
V_y, m/s	-3.0	-1.0	1.0	2.0	3.0	*x*, m	0.0	230	410	570	-
						V_y, m/s	-2.8	-1.1	1.2	1.7	3.4

The convolution integral (6) for the intensity in the telescope focal plane allows the relationship

$$I(F_t, \rho) = \int d\kappa\, \tilde{I}_l(x, \kappa)\, \tilde{I}_T(x, \kappa, \rho)\, e^{i\kappa\rho}\,, \qquad (7)$$

where

$$\tilde{I}_l(x, \kappa) = (2\pi)^{-2} \int d\mathbf{r}\, I_l(x, \mathbf{r})\, e^{-i\kappa\mathbf{r}}\,,$$

$$\tilde{I}_T(x, \kappa, \rho) = (2\pi)^{-2} \int d\mathbf{r}\, I_T(x, \mathbf{r}, \rho)\, e^{-i\kappa\mathbf{r}}\,, \qquad (8)$$

to be applied for calculation of the intensity $I(F_t, \rho)$ with use of the fast Fourier transform. Equations (7), (8) provide calculation of the 2-D intensity distributions of reflected optical wave in the focus of receiving telescope with acceptable computer consumptions.

Testing of the algorithm for reconstruction of wind profile for the model of random phase screen was performed by the following way. Propagation of the collimated illuminating Gaussian beam (beam radius $a = 5 \cdot 10^{-2}$ m, focal length $F^{-1} = 0$, path length $x = 1$ km, wavelength $\lambda = 5 \cdot 10^{-7}$ m) and the telescope beam (beam radius $a_t = 3 \cdot 10^{-2}$ m, focal length $F_t = 1$ m) was simulated by the algorithm[9]. The single random phase screen located at the beginning of the path and moved with the speed of $V_z = 3$ m/s (*Z*-direction) and $V_y = -1.5$ м/с (*Y*-direction) was simulated. The computational grid had 512 × 512 mesh points with a distance between the points of $5 \cdot 10^{-4}$ m. The simulated distributions of intensity of the illuminating and the telescope beams in the reflector plane were Fourier-transformed, then the Fourier transformations were multiplied and the inverse Fourier transform was performed. The obtained intensity distributions in the telescope focal plane were used to calculate the spatio-temporal correlation function (1). The series of 256 frames of 2-D intensity distributions were processing. The correlation function was estimated from 10 realizations of such series. Then the correlation function was Fourier-transformed over the coordinate ρ and time τ. Logarithm of the spatio-temporal spectrum calculated in that way is presented in Fig. 3.

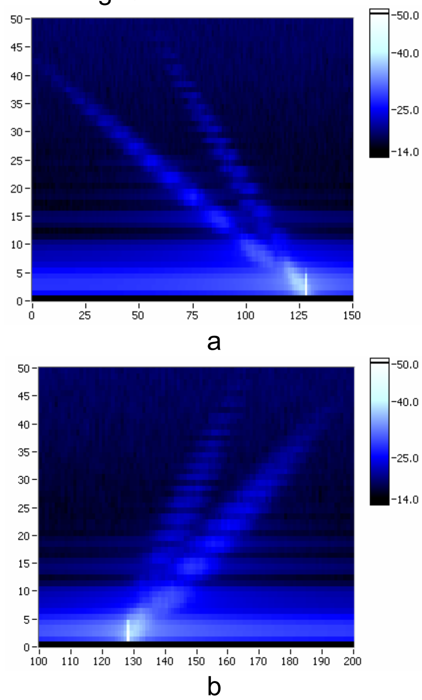


Figure 3. Single screen spectrum. a – *Z*-component, b – *Y*-component.

The temporal frequency ω is laid off abscissa axis and the spatial frequencies q_z (Fig. 3a) and q_y (Fig. 3b) are laid off ordinate axis.

As would be expected from the theoretical analysis of the problem in more general formulation than above (incident spherical wave and infinite scattering surface), the calculated spectra in Fig. 3 have two inclined bands for every coordinates of the spatial frequency **q** with the sharply defined oscillations. The reconstruction results for the component of wind velocity V_z = 3 m/s from two bands in Fig. 3a are following: the screen coordinate is of x = 20 m; x = 10 m; the wind velocity is of V_z = 2.9 m/s; V_z = 3.0 m/s. For the component V_y = –1.5 m/s we have x = 10 m; x = 0.0 m; V_y = –1.3 m/s; V_y = –1.6 v/s. That is, the every band of spectrum allows the position and transverse speed of screen to be determined to a high accuracy.

Thus, based on the above results and the results of other end-to-end computer experiments we can conclude that wind profiling from the scintillations of reflected optical wave is possible.

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