Dispelling the Myth of Reduced Heterodyne Efficiency With Increasing Detection Aperture

George E. Busch[1] and Diego F. Pierrottet
Coherent Applications, Inc.
101 Research Drive Suite 101-C
Hampton, Virginia 23666
jorge@coherent-app.com

I. INTRODUCTION

Heterodyne detection offers the possibility of greatly improved detection sensitivity relative to direct detection techniques. As has often been cited[2], the possibility of sensitivities down to single photon detection is, in principle, achievable. This is primarily because it is possible to narrow the effective bandwidth of the optical signal down to ~ 1 MHz, or less, by standard RF filtering techniques on the intermediate frequency (IF) signal. Achieving such narrow bandwidths would be an extremely difficult task by strictly optical techniques. As a result, the total noise can be restricted to just the shot-noise in the cw local oscillator (LO) current. Since the heterodyne signal is also proportional to the LO intensity, the carrier-to-noise ratio (CNR) becomes independent of the LO intensity when the noise is completely LO shot-noise limited.

However, for coherent detection of signals that arise from reflection of coherent beams from diffuse surfaces, or from dispersed atmospheric aerosol particles, there is an issue that arises because the amplitude and the phase of the signals are only correlated over limited transverse distances within the scattered radiation field. Thus the usual assumption is that the apertures of receivers must be restricted to gather only signals that are correlated. This approach appears to be supported by antenna theory[3] and by numerous calculations of expected heterodyne efficiency vs. aperture size[4], but very few actual experiments[5], to test the assumption have been reported.

In the present paper, we call into question the validity of the assumption that heterodyne efficiency falls as aperture area is increased for speckle radiation fields. We suggest that for appropriately chosen LO parameters, the heterodyne efficiency can remain high, and CNR can increase with aperture. If this is the case, it is a very important contribution to laser radar field, because the detection sensitivity and range presently being achieved with apertures that have only single speckle dimensions can be significantly improved by an increase in the receiver aperture.

II. SPECKLE PROPERTIES

In the usual approach[4] to arriving at a maximum heterodyne efficiency, the signal from a coherent source, such as a uniform plane wave or a single mode Gaussian, is calculated to determine the maximum possible signal. Then the fraction of the total scattered signal intercepted by the receiver aperture that is coherent, i.e. within a single speckle, is calculated, and ratioed to the total received signal, to arrive at the efficiency. We suggest that this approach may not properly provide for the way that circular complex Gaussian random signals should be summed, and thus we take an alternate approach, described next.

Fig.1 shows a typical polarized speckle pattern that might be encountered at the receiver, for the optical signal in a scattering experiment. The probability density for the intensity at each point in the pattern obeys negative exponential statistics[6], and is given by

$$p_I(I) = \frac{1}{\langle I \rangle} \exp\left(-\frac{I}{\langle I \rangle}\right), \qquad (1)$$

in which $\langle I \rangle$ is the average intensity over the distribution, i.e.

$$\langle I \rangle = \int_0^\infty I \bullet p_I(I)\, dI \qquad (2)$$

But, the most probable intensity at every point (highest probability density) given by Eqn.(1) is zero. Two overlays of receivers are also shown in the figure, with apertures approximating the dimensions of a single speckle. Thus, while the overlay of the aperture labeled A, in Fig.1, shows a high intensity at the center of the aperture, the aperture labeled B, has low intensity near the center. In view of the negative exponential distribution, a single realization of low intensity, such as encountered by aperture B, has relatively high probability. There will be a corresponding speckle distribution at the detector plane, with the same number of speckles (~1, for receiver aperture the order of a single correlation area) and with equal integrated intensity, but reduced by vignetting and transmission losses of the receiver optical components.

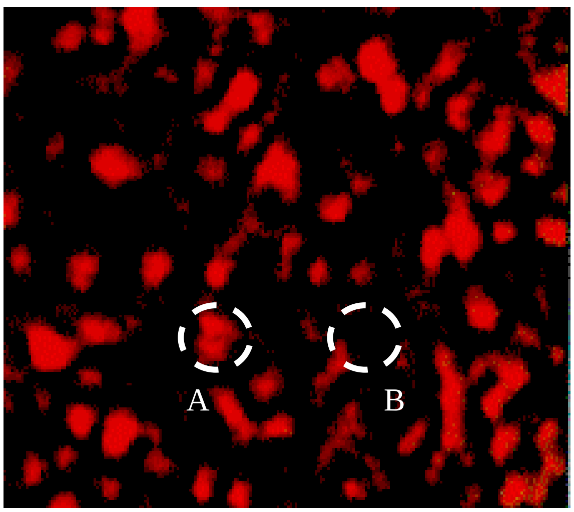

Fig. 1. Typical laser speckle pattern, with two overlaid apertures having areas comparable to a correlation area. Aperture A happens to coincide with an intensity maximum; however, Aperture B is more typical, with a most probable intensity at any point being close to zero.

III. HETERODYNE IF CURRENT

When the signal (S) intensity distribution is combined with the LO distribution on the detector, a current density will be generated at each point, given by

$$J(x,y,t) = J_{LO}(x,y) + J_S(x,y) + 2\,j_{LOS}(x,y,t) \quad (3)$$

in which $J_{LO}(x,y)$ is the constant current density generated by the LO, $J_S(x,y)$ is the constant current density generated by S, and $\Delta\omega$ is the intermediate frequency (IF) offset between the S and LO optical frequencies. $\Delta\phi$ is the difference in phases of the LO and S, and thus is the phase of the IF signal, and j_{LOS} (x,y,t) is the alternating current density at frequency $\Delta\omega$

$$j_{LOS}(x,y,t) = J_{LOS}(x,y)\cos(\Delta\omega t + \phi(x,y)) \quad (4)$$

If we assume the LO has the same phase everywhere on the detector surface, then without loss of generality, we can set it to zero, and $\Delta\phi$ is the phase of S.

Each of the current densities on the right-hand side of Eqn.(3) is proportional to both the local detector quantum efficiency, $\eta(x,y)$, and the charge of an electron, e, and each is inversely proportional to the photon energy $h\nu$, so that

$$J_\beta(x.y) = \alpha(x,y) I_\beta(x,y)$$
$$\alpha(x,y) = \eta(x,y)(e/h\nu) \quad (5)$$
$$\beta = LO, S, or\ LOS\ .$$

I_{LO} *and* I_S are the intensities of the LO and S fields, and I_{LOS} is the product of the root-mean-square amplitudes of the two optical fields

$$I_{LOS}(x,y) = (I_{LO}(x,y) I_S(x,y))^{1/2} \quad (6)$$

Thus, anywhere I_{LOS} is low, i.e. either the intensity of LO, the intensity of S, or both, are low, then the IF current density and the corresponding contribution to the integrated IF current will also be small. Furthermore, for uniform intensity and phase of LO, and uniform quantum efficiency[7], the current density at any point is directly proportional to the amplitude of S, but with frequency $\Delta\omega$

$$\begin{aligned} j_{LOS}(x,y,t) &= \alpha I_{LOS}(x,y)\cos(\Delta\omega t + \phi_S(x,y)) \\ &= \alpha^\circ I_S^{1/2}(x,y)\cos(\Delta\omega t + \phi_S(x,y)) \end{aligned} \quad (7)$$

The total IF current amplitude $\mathcal{J}_D(t)$, measured at the output terminal of the detector is then obtained by integrating j_{LOS} (x,y,t) over the area A_D of the detector

$$\mathcal{J}_D(t) = 2\iint_{A_D} j_{LOS}(x,y,t)\,dxdy \quad (8)$$

In the next sections we consider how to do the appropriate integration of Eqn. (8) to arrive at the IF alternating current from the area of the detector, using the random walk model to allow for the speckle distributions of amplitude and intensity of S.

IV. RANDOM WALK STATISTICS

Since space is limited for this summary article, we point out the similarities that the present problem of adding signals with random phase and amplitude has with the standard problem of a random walk, in which each step of the walk has equal probability of going in any direction. We then utilize what is known about statistics and probability functions of the variables to derive the results we expect for heterodyne detection. Since Goodman[6] used the same approach to derive properties of speckle fields, and since speckle fields are the starting points for our heterodyne problem, we tailor our treatment to his.

The amplitude at each point in a speckle pattern results from summing uncorrelated scattered light contributions from a large number of points on the scattering surface. Using a model like the one shown in Fig. 2, Goodman[6] shows that the amplitude of the light at each point in the speckle pattern is a zero mean Gaussian random variable. The current

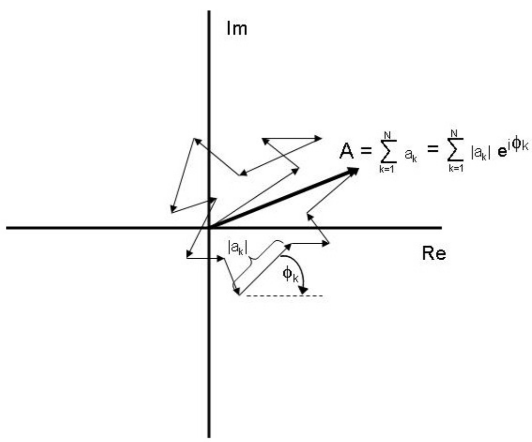

Fig.2. Schematic random walk model for the summation of many complex phasors, a_k, to obtain the complex phasor A, a zero mean Gaussian random variable, representing the optical field at a point in a speckle pattern.

density at each point on the detector, (see Eqn.(7)) is also proportional to the signal light amplitude, thus it is a zero mean Gaussian random variable, and since the sum of (or integral over) any number of zero mean Gaussian random variables is also a zero mean Gaussian random variable, the total current, summed at the detector output terminal, is a zero mean Gaussian random variable.

Separating the components of the complex phasor A into real and imaginary parts

$$A = A^{(r)} + iA^{(i)} = \sum_{k=1}^{N} a_k = \sum_{k=1}^{N} |a_k| \exp(i\phi_k) \qquad (9)$$

Employing the central limit theorem for large N, Goodman[6] derives the joint probability density function for $A^{(r)}$ and $A^{(i)}$

$$p_{r,i}(A^{(r)}, A^{(i)}) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{[A^{(r)}]^2 + [A^{(i)}]^2}{2\sigma^2}\right) \qquad (10)$$

with

$$\sigma^2 = \lim_{N\to\infty} \sum_{k=1}^{N} \frac{\langle |a_k|^2 \rangle}{2} \ . \qquad (11)$$

For the intensity at a point $I(x,y)$, given by

$$I(x,y) = |A(x,y)|^2 = [A^{(r)}]^2 + [A^{(i)}]^2$$
$$A^{(r)} = (I)^{1/2} \cos\phi \qquad A^{(i)} = (I)^{1/2} \sin\phi \ , \qquad (12)$$

Goodman[6] derived the negative exponential probability density for $I(x,y)$ in Eqn.(1). The mean value of $I(x,y)$ is

$$\langle I(x,y) \rangle = 2\sigma^2 \ . \qquad (13)$$

V. IF CURRENT RANDOM WALK

Next we look at the correspondences between the variables $A(x,y)$ and $I(x,y)$ at the point (x,y) in the speckle pattern, and the corresponding variables related to heterodyne IF currents. Referring to Eqn.(8), we utilize the complex variable formalism (see, e.g., Yariv[8]) and write the $j_{LOS}(x,y,t)$ as the real part of the analytic signal $V_{jLOS}(x,y,t)$ of $j_{LOS}(x,y,t)$

$$j_{LOS}(x,y,t) = \mathrm{Re}[V_{jLOS}(x,y,t)]$$
$$V_{jLOS}(x,y,t) = j_{LOS}(x,y) \exp(i\Delta\omega t) \qquad (14)$$
$$j_{LOS}(x,y) = |j_{LOS}(x,y)| \exp(i\phi(x,y)) \ ,$$

with $j_{LOS}(x,y)$ being a complex phasor with amplitude from Eqns.(4) and (7),

$$|j_{LOS}(x,y)| = J_{LOS}(x,y) = \alpha^{\circ} I_S^{1/2}(x,y) \qquad (15)$$

We assert that $j_{LOS}(x,y)$ is then a zero mean Gaussian random variable, and being integrated over the detector area, and multiplied by the constant α^{o}, it remains a zero mean Gaussian random variable, and thus

$\mathcal{J}_D(t)$, obtained from Eqn.(8), is a zero mean Gaussian random variable.

Finally, we square the IF current output from the detector to get the alternating current power

$$P_D = \left|\mathcal{J}_D(t)\right|^2 \quad (16)$$

Since $\mathcal{J}_D(t)$ is a zero mean Gaussian random variable, summed at a single point, (that point being the output terminal of the detector), we expect P_D to follow the statistics obtained by Goodman[6] for $I(x, y)$, the intensity at a single point, i.e., a negative exponential distribution. Thus, the most probable power measured at the IF frequency would be zero, but the average would be obtained by integrating the square of the current densities given by Eqn.(7), over the area of the detector. In correspondence with the results for intensity at a single point, given by Eqns.(9) and (11), for discrete sums, the average power for integrated currents is

$$\begin{aligned}\langle P_D\rangle &= \langle\left|\mathcal{J}_D(t)\right|^2\rangle \\ &= \langle[\iint_{A_D} j_{LOS}(x, y)dxdy]^2\rangle \quad (17) \\ &= \iint_{A_D}\langle j^2_{LOS}(x, y)\rangle dxdy \ ,\end{aligned}$$

and thus the average power in the integrated IF current signal is equal to the integral of the average power density of the current over the surface of the detector.

Substituting the definitions for α relating current to optical power from Eqn.(5), into $\langle j^2_{LOS}(x, y)\rangle$, and integrating, the average heterodyne power is

$$\langle P_D\rangle = \alpha^2 P_{LO}\langle P_S\rangle \quad (18)$$

in which P_{LO} and P_S are the optical powers onto the detector. Since

$$\langle P_S\rangle = \langle I_S\rangle_R A_R \ , \quad (19)$$

i.e., the average power of S at the detector is proportional to average intensity at the receiver times the area of the receiver aperture. Thus the average heterodyne signal power is proportional to the area of the receiver aperture, as asserted at the outset of the forgoing analysis.

VI. CONCLUSION

We have shown that increasing the aperture of the receiver in an optical heterodyne measurement of returns from diffuse scattering targets, is equivalent to increasing the number of steps in a random walk problem. The carrier-to-noise-ratio (CNR) in the heterodyne measurement, being equal to the number of signal photons detected, increases in direct proportion to the area of the receiver, just as the mean square distance from the origin for a of a particle in the random walk problem increases linearly with the number of steps taken. Thus, the often quoted conclusion that the receiver aperture should be limited to a single coherence area of the speckle field appears to be incorrect. It should be noted, however, that in many applications of interest, it is likely that signal-to-noise ratio (SNR) is the important consideration, rather than CNR. In view of the fact that SNR has a maximum value of unity, and is insensitive to aperture until CNR drops to close to unity, careful consideration must be given to determine whether the increased CNR from larger receiver apertures is worth the penalties in system weight and size.

REFERENCES

---

[1] Work by GEB was done at Los Alamos National Laboratory, Los Alamos, NM 87545.
[2] A. Yariv, Optical Electronics, Fourth Edition, Saunders College Publishing, Holt , Rinehart, and Winston, Inc, 1991, pp. 407-11.
[3] A.E. Siegman, "The antenna properties of heterodyne receivers", Appl. Opt. 5(1966)1588-94.
[4] W.R. Leeb, P.J. Winzer, and K.H. Kudielka, "Aperture dependence of the mixing efficiency, the signal-to-noise ratio, and the speckle number in coherent lidar receivers", Appl. Opt. 37(1998)3143-8.
S.C. Cohen, "Heterodyne detection: phase front alignment, beam spot size, and detector uniformity", Appl.Opt. 14(1975)1953-9.
Y. Zhao, M.J. Post, and R.M. Hardesty, "Receiving efficiency of monostatic pulsed coherent lidars. 1: Theory", Appl. Opt. 29(1990)4111-9.
[5] J.Y. Wang, B.J. Bartholomew,, M.L. Streiff, and E.L. Starr", Imaging CO2 laser radar field tests" Appl.Opt. 23(1984)2565-71.
[6] J.W. Goodman, "Statistical Properties of Laser Speckle Patterns" , in Laser Speckle and Related Phenomena, edited by J.C. Dainty, Topics in Applied Physics v.9, Springer Verlag, Berlin, 1984.
[7] Uniformity of LO amplitude and phase, as well as surface quantum efficiency is not required, but simplifies the present treatment.
[8] Reference 2, pp. 1-2.