Dead-Time Effects on Geiger-Mode APD Performance

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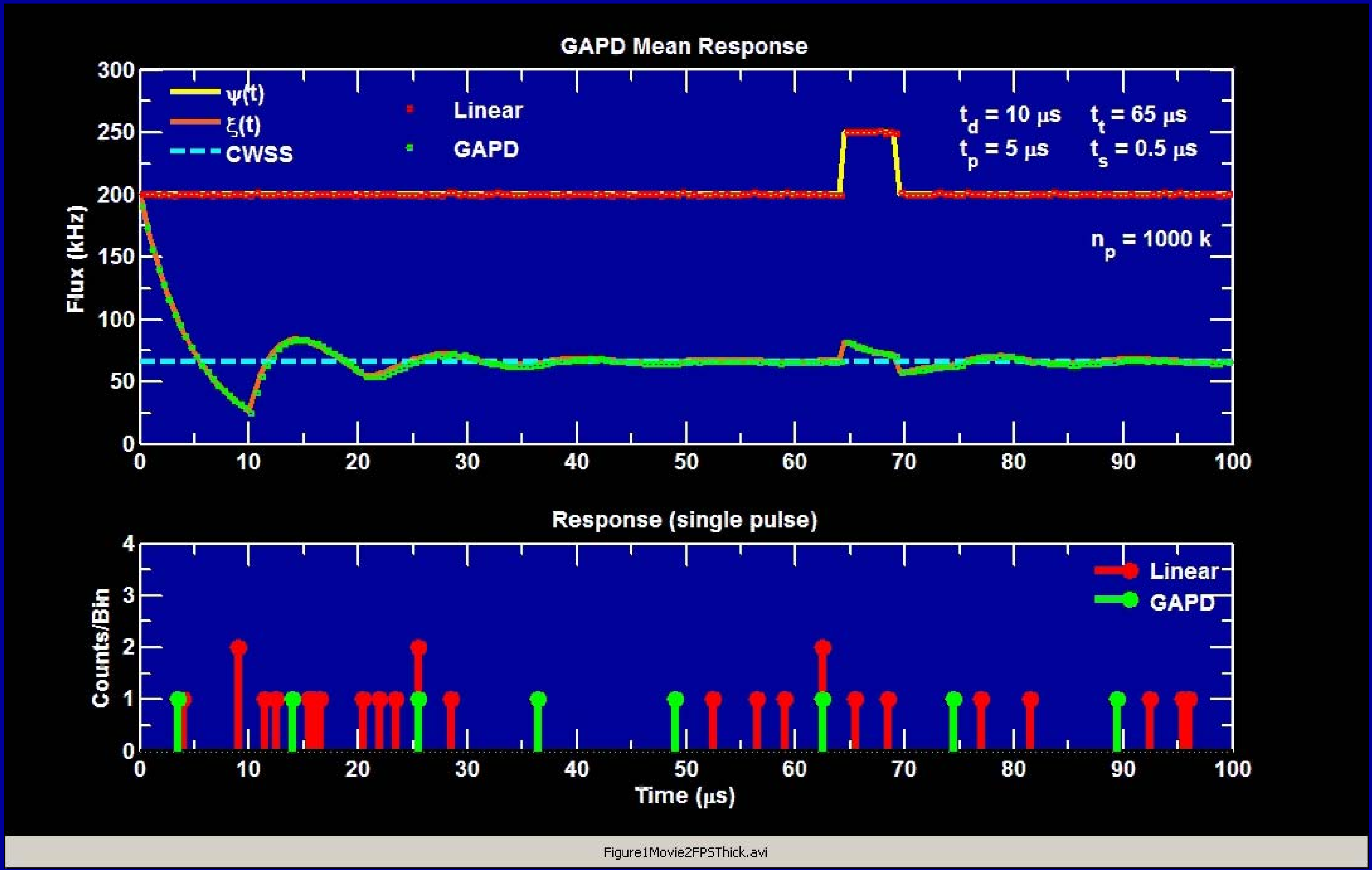
Simulated Linear and Geiger Mode Signals

- Dead Time: 10 us
- BG Flux: 200 kHz
  - 2/dead time
- Pulse Length: 5 us
- #signal photons/pulse: 0.25



Outline

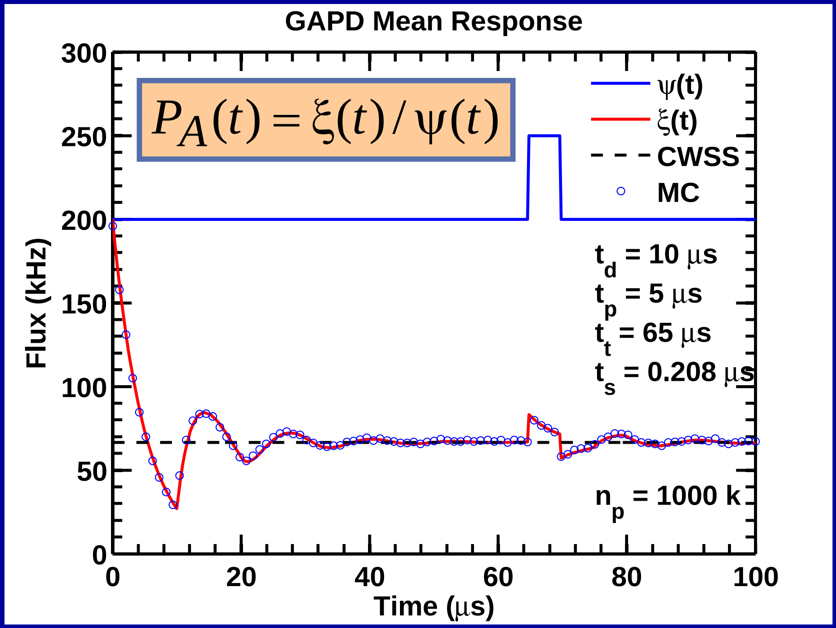
- Problem Overview and Assumptions
- Response Theory
- Efficiency Theory
- SNR Theory
- Detection Statistics
- Advantages of Micro-detectors
- Summary/Conclusions

Problem Overview

- **GAPD Model**
  - Linear front end, PDE or QE
  - High gain region => saturation
  - Detects only 1 event per dead time
    - Quench (reset) and trap (after pulses) time drive dead time
- **Signal Model PE Model**

$$\psi(t) = \psi_s(t) + \psi_c(t) + \psi_{bg} + \psi_d$$
$$= \eta_q (P_s(t) + P_c(t) + P_{bg} + P_d) / h\nu$$

- **Assumptions**
  - Poisson noise statistics
  - Negative Binomial signal statistics
  - Short pulse
    - Tp < Td, so at most one detection per matched filter width
  - Multi-pulse Accumulation
    - Matched Filter Receiver
      - # noise PEs = m_n < 1
      - # signal PEs = m_s



Probability that the GAPD is Active, $P_A(t)$, is the ratio of the mean-output flux to the input flux

Poisson Properties

- **Memoryless**
  - **probability of an event in a given time interval is independent of previous events**
- **Distribution**

$$\Pr(k;t_1,t_2) = \frac{e^{-m(t_1,t_2)}m(t_1,t_2)^k}{k!}$$

$$m(t_1,t_2) = \int_{t_1}^{t_2}\psi(t)dt$$

- **Probability of no event**

$$\Pr(k=0;t_1,t_2) = e^{-m(t_1,t_2)}$$

- **Probability of one or more events**

$$\Pr(k>0;t_1,t_2) = 1 - e^{-m(t_1,t_2)}$$

- **Event time relative to a fixed time, t_o, for constant flux, ψ_o**
  - **1st event time is described by an exponential PDF**

$$f_1(t-t_o) = \psi_o \exp[-\psi_o(t-t_o)]$$

  - **nth event time is described by a Gamma PDF**

$$f_n(t-t_o) = \psi_o^n (t-t_o)^{n-1} \exp[-\psi_o(t-t_o)]/(n-1)!$$

Mean Response Theory: General Flux

- Total mean response is sum of n^{th} event mean responses

$$\xi(t) = \sum_{n=1}^{N_d(t)} \xi_n(t)$$

- First event mean response
  - Exponential PDF
  - Exponential Rate

$$\Pr(k > 0; 0, t) = 1 - e^{-m(0,t)}$$

$$\xi_1(t) = \psi(t) e^{-m(0,t)}$$

- 2^{nd} event mean response, given first event

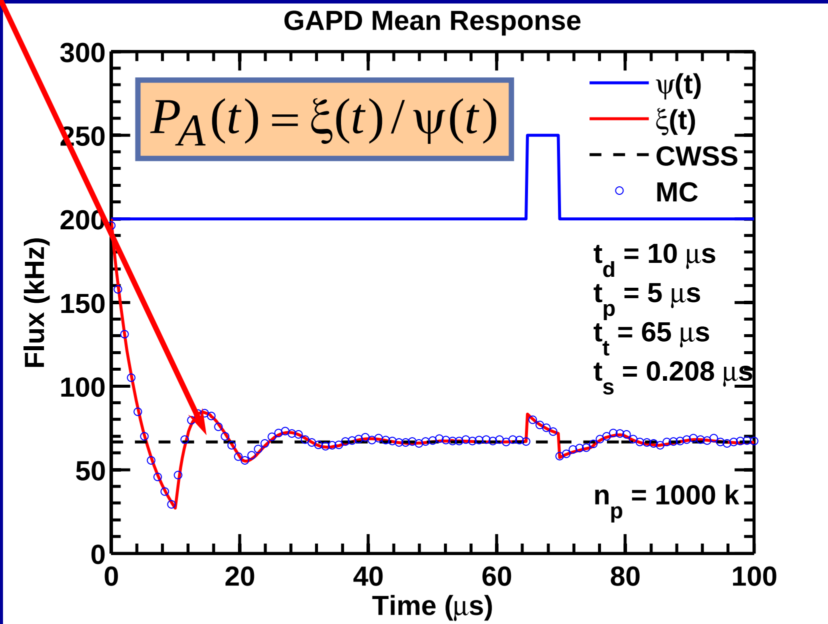
$$\xi_2(t|t_1) = \psi(t) e^{-m(t_1+t_d, t)}$$

- 2^{nd} event mean response

$$\xi_2(t) = \int_0^{t-t_d} \xi_1(t_1)\xi_2(t|t_1)dt_1 = \psi(t)\int_0^{t-t_d} \xi_1(\tau)e^{-m(\tau+t_d,t)}d\tau$$

- n^{th} event mean response

$$\xi_n(t) = \psi(t) \int_{t_d(n-2)}^{t-t_d} \xi_{n-1}(\tau)e^{-m(\tau+t_d,t)}d\tau$$



Mean Response Theory: Step Function Input Flux

- **The general theory reduces to a sum of delayed Gamma PDFs**

$$\xi_1(t) = \psi_o e^{-\psi_o t}$$

$$\xi_n(t) = \psi_o^n \left[t-(n-1)t_d\right]^{n-1} e^{-\psi_o\left[t-(n-1)t_d\right]}/(n-1)!$$

  - **Mean = $(n\text{-}1)t_d + n/\psi_o$ and diversity parameter, n**
- **CWSS ($m_d = \psi_o t_d$)**
  - **Mean time between detections:**

$$E[\Delta T] = (1/\psi_o + t_d) = (1+m_d)/\psi_o$$

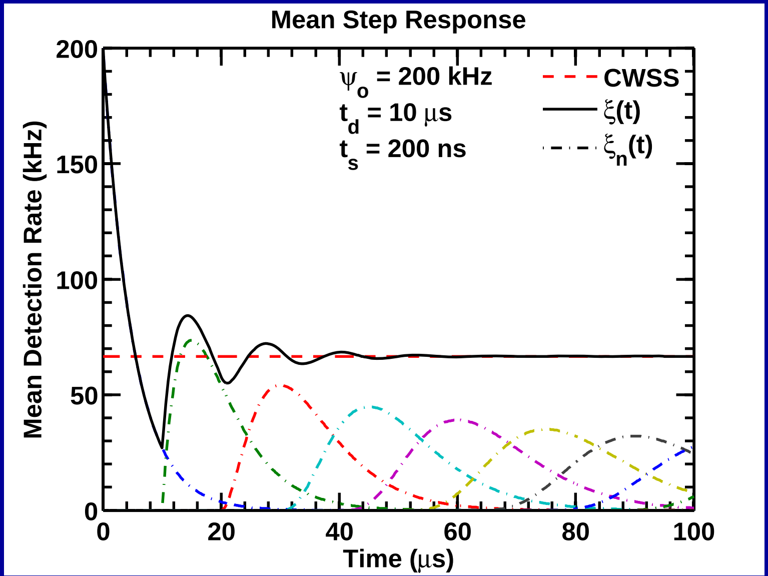
  - **Mean detection rate:**

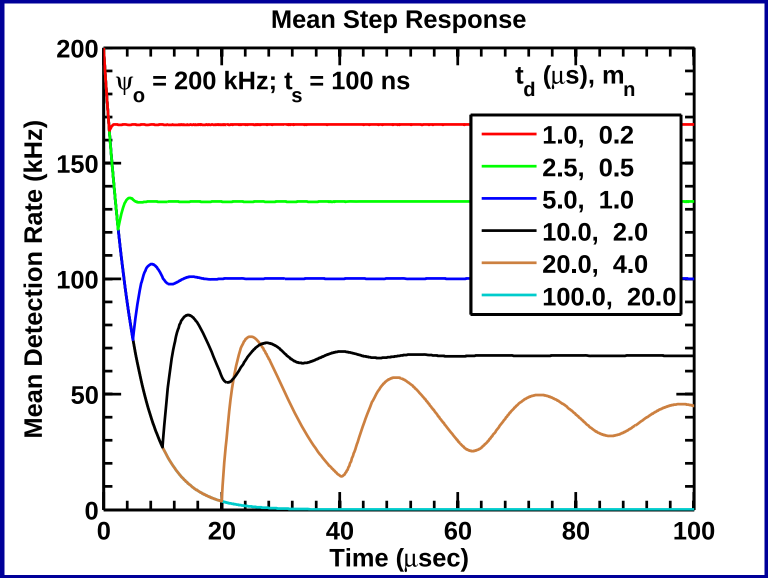
$$\xi_{cwss} = 1/(1/\psi_o + t_d) = \psi_o/(1+m_d)$$

  - **Probability Active:**

$$P_{A_{cwss}} = 1/(1+m_d)$$

- **Infinite sum (over diversity) of Gamma PDFs is constant**
  - **ψ_o for zero dead time**
  - **$\psi_o/(1+m_d)$ for non-zero dead time**
  - **CWSS time scales as $\psi_o t_d^2$**





Signal Photon Detection Efficiency (SPDE)

- **Signal + noise PE Probability of detection**

$$PD_m = P_A(t_t)(1 - e^{-m_s - m_n})$$

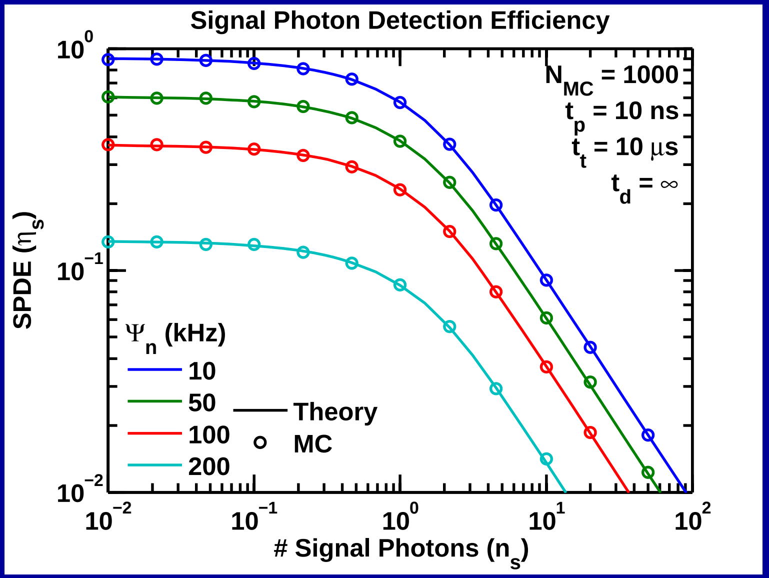
- **Signal PE Probability of detection**
  - **If m_n << 1 then signal and noise detection probabilities are equal and governed by the signal strength**

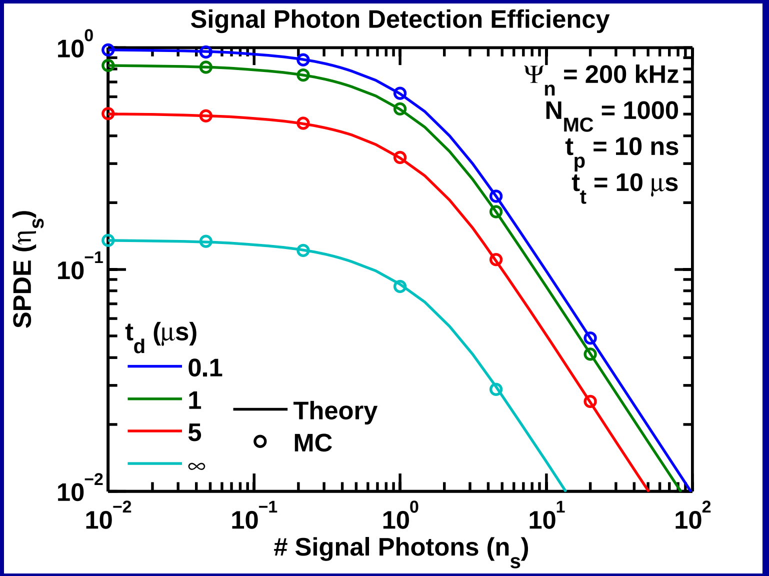
$$PD_s \approx P_A(t_t)(1 - e^{-m_s})$$

- **Efficiency**

$$\eta_s = PD_s / n_s$$

$$\eta_s = P_A(t_t)(1 - e^{-m_s}) / n_s$$





Signal-to-Noise Ratio

- **SNR Definition**

$$SNR = \frac{E[s]^2}{\mathrm{var}[s+n]}$$

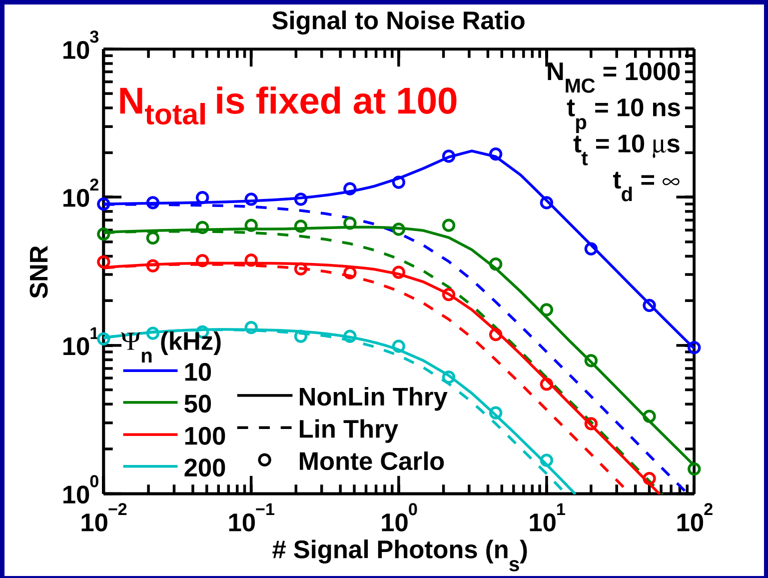
- **Linear Device (Np pulses)**

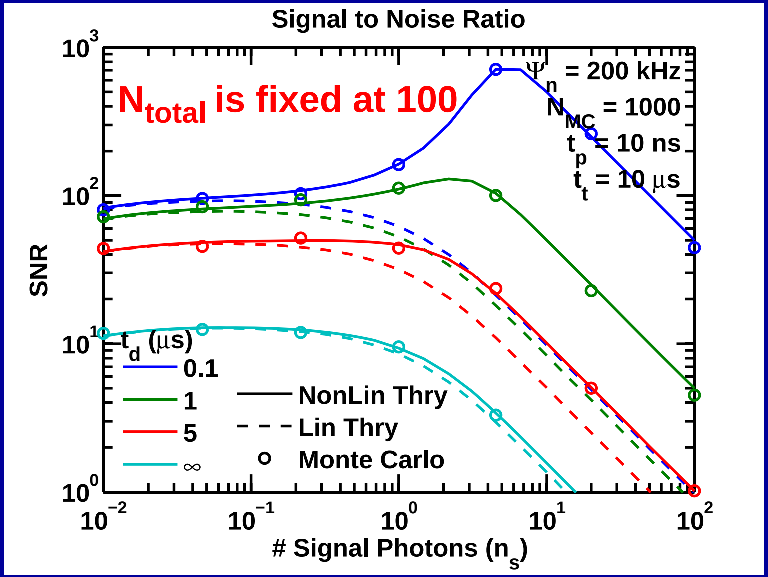
$$SNR = \eta_s N_p \frac{n_s^2}{n_s + n_n}$$

- **Non-linear model**
  - Assume Binomial Distribution
  - Np trials
  - Mean = $N_p * p_s$; $p_s = P_A(1-\exp[-m_s])$
  - Var = $N_p * p_{sn}(1-p_{sn})$; $p_{sn} = P_A(1-\exp[-m_s-m_n])$

$$SNR \approx \frac{N_p P_A \left(1-e^{-m_s}\right)^2}{\left(1-e^{-m_s-m_n}\right)\left(1-P_A\left(1-e^{-m_s-m_n}\right)\right)}$$

- **Deviations from linear model**
  - Device is behaving non-linearly
  - Detections are not Poissonian
  - Little reflectance information is available





GAPD Detection Statistics: Overview

- **Assumption**
  - **Newman-Pearson detection problem**
    - **Threshold crossing problem with Pd and Pfa per matched-filter bin**
  - **Multi-pulse accumulation**
    - **Constitutes a Bernoulli trial (1 of 2 outcomes per pulse, per range-gate)**
    - **Binomial distribution analysis applies for noise and signal+noise statistics**

- **Binomial distribution describes the PDF of the number of successes that occur in *n* Bernoulli trials, where the probability of success in a trial is, *p*.**

$$p(k;n) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

- **Detection statistics (Pd or Pfa, denoted Pr below) are equal to the tail of a binomial CDF, which has support up to N_p pulses accumulated**

$$\Pr = \sum_{k=k_{th}}^{N_p} p(k;N_p) = 1 - \sum_{k=0}^{k_{th}-1} p(k;N_p)$$

  - **Where k^{th} is a threshold needed to achieve a Pfa specification**

GAPD Detection Statistics: Pd and PFA

- **For GAPDs, an appropriate expression for the single trial (1 pulse) probability "p" for noise (Pfa) or signal + noise (Pd) is needed to substitute into the binomial distribution**

- **For the noise, Poisson statistics apply. The prob of at least 1 detected PE is**

$$p_n = \Pr[k_n > 0] = P_A\left(1 - e^{-m_n}\right)$$

  - **m_n is the number of noise PEs in a matched filter integration time**
  - **P_A is the probability that the GAPD is armed**

- **For negative binomial speckled-signal plus Poisson noise, Goodman's expression with arbitrary diversity, *M*, (spatial, spectral and polarization) applies: Evaluation of this expression for zero PE's leads to the following simple expression**

$$p_{sn} = \Pr[k_{sn} > 0] = P_A\left(1 - e^{-m_n}\left(1 + m_s / M\right)^{-M}\right)$$

$$\lim_{M \to \infty}\left(1 + m_s / M\right)^{-M} = e^{-m_s}$$

  - **m_s is the number of signal PEs in a matched filter integration time**
  - **Glint targets are handled by taking the infinite M limit**

GAPD Detection Statistics: Detail Math BACKUP
Noise and Signal + Noise Statistics

- **Noise : Poisson PDF and Prob m > 0**
  - **m_n is mean number of noise PEs in a matched filter bin**

$$p_n(k) = m_n^k \exp(-m_n)/k!$$

$$\Pr[k_n > 0] = 1 - p_n(0) = 1 - \exp(-m_n)$$

- **Signal + Noise PDF (NegBinomial + Poisson), See Goodman**
  - **M is the diversity, m_s is mean number of signal PEs**

$$p_{sn}(k) = \left(\frac{M}{M+m_s}\right)^M \frac{e^{-m_n}}{(M-1)!} \sum_{j=0}^{k} \frac{(k+M-j-1)!}{j!(k-j)!}(m_n)^j \left(\frac{m_s}{M+m_s}\right)^{k-j}$$

$$\Pr[k_{sn} > 0] = 1 - p_{sn}(k=0) = 1 - e^{-m_n}\left(1 + m_s/M\right)^{-M}$$

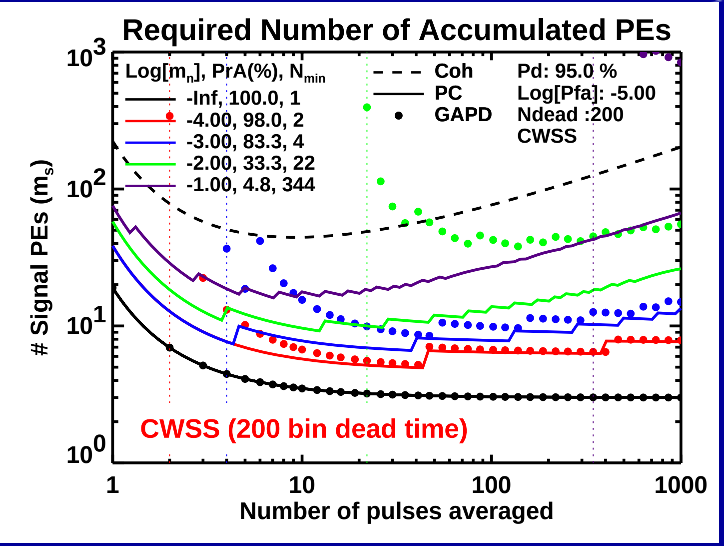
  - **High diversity limit (Boise Einstein)**

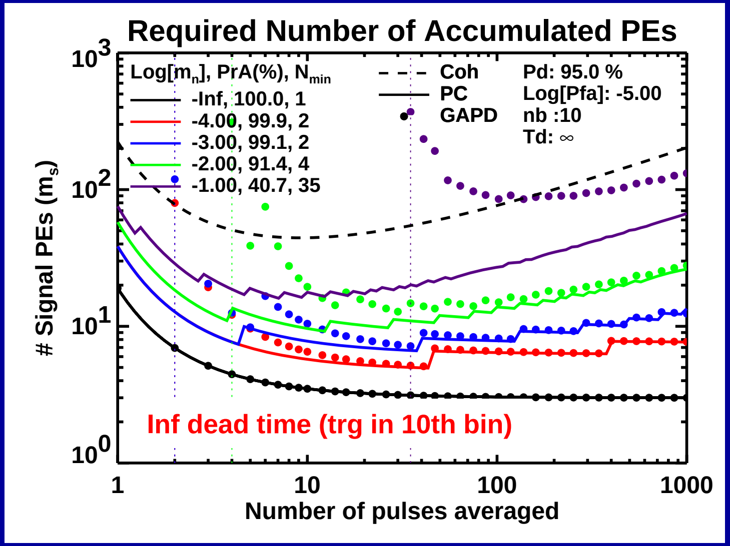
$$\lim_{M \to \infty} \left(1 + m_s/M\right)^{-M} = e^{-m_s} \quad \Rightarrow \quad p_{sn}(k>0) = 1 - e^{-m_s - m_n}$$

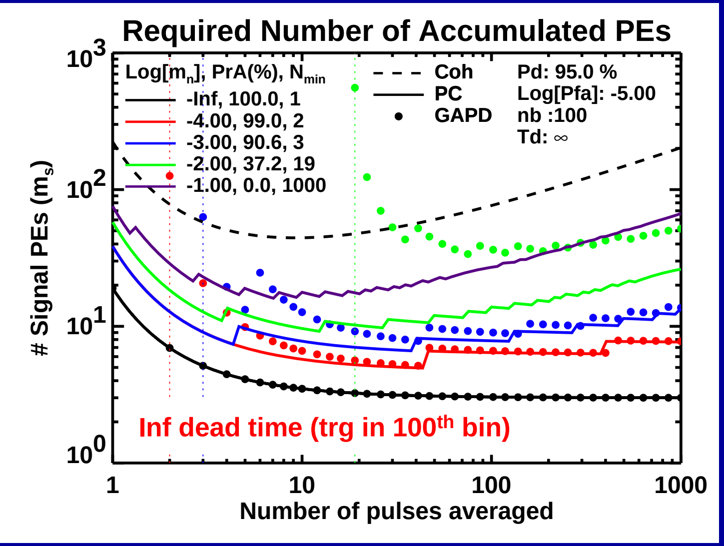
- **These probabilities then need to be multiplied by P_A , the probability that the GAPD is active.**

Performance Comparison: Normalized to Matched Filter Bin Width

- **Example Case**
  - 10ns/bin
  - ψ_o = 100 kHz -> m_n = 0.001
- **CWSS example**
  - 2 us dead time -> 200 bins
- **Long dead time example**
  - 10^{th} bin -> 15 m gate-to-target range
  - 100^{th} bin -> 150 m gate-to-target range

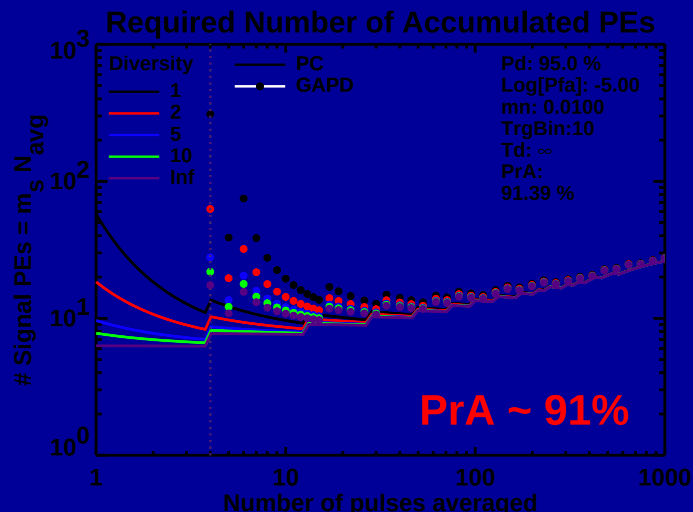


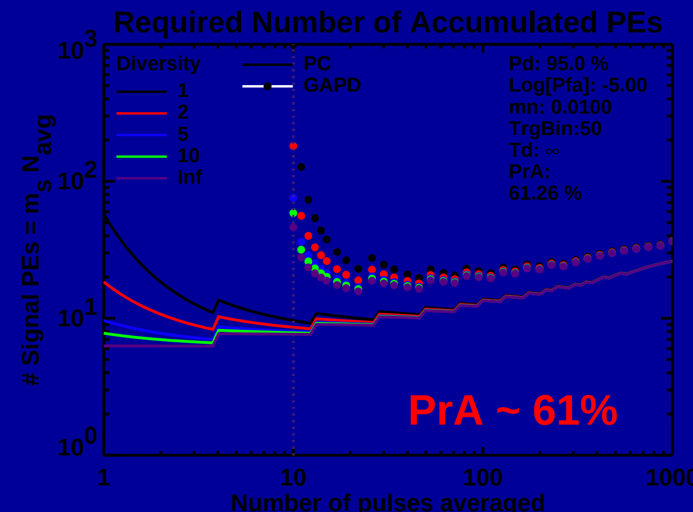


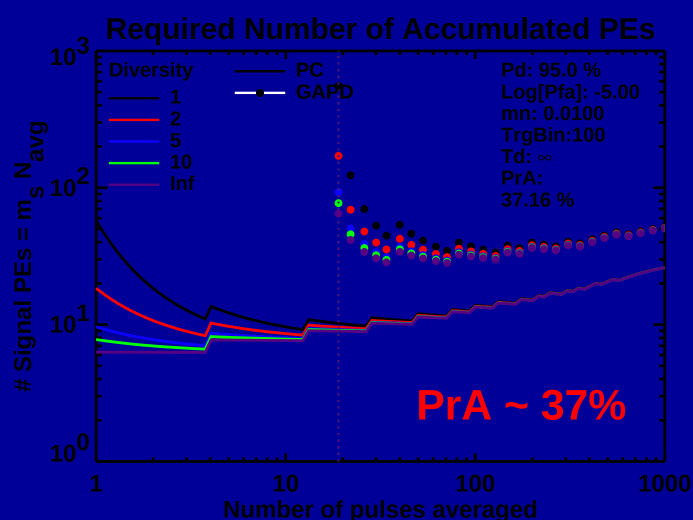


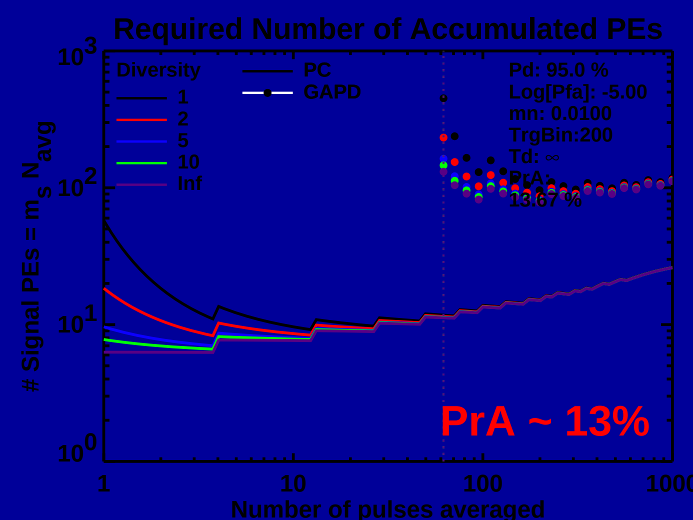
Diversity Trades:

- **Key parameters**
  - **Pd = 95%**
  - **Pfa = 10^{-5}**
  - **Varying PrA**









Advantage of Micro-pixels

- One approach to increasing SPDE is to combine signals from multiple detectors to form a single pixel

- Net SPDE (General illumination)
  - Weighted sum of micro-pixel efficiencies
  - Weights are fraction of total # signal photons

$$\eta_s = \sum_{i=1}^{N_d} w_i \eta_{s_i} \quad \text{where} \quad w_i = n_{s_i} / n_s$$

- Net SPDE (Uniform illumination)
  - Weights are equal (1/Nd)
  - Net SPDE is average of micro-pixel SPDEs, which can be much higher than would be achieved with one large pixel

$$\eta_s = (1/N_d) \sum_{i=1}^{N_d} \eta_{s_i}$$

Summary/Conclusions

- **Derived Expression for the Mean GAPD Output Flux**
  - Response (flux) is sum of individual event response functions or PDFs
    - Recursive theory for the nth event PDFs was developed
    - For the special case of a constant input flux these PDFs are delayed Gamma PDFs with mean $(n\text{-}1)t_d+n/\psi_o$ and diversity, n
  - CWSS response was derived
- **P_A = Probability that GAPD is active**
  - Shown to be ratio of output to input flux
  - This probability depends on number of noise PEs prior to target within a dead time
  - Shorter dead time devices have increased P_A
- **Signal Photon Detection Efficiency derived and shown to agree with MC measurements**
  - Good efficiency requires
    - High P_A
    - $m_s < 1/(1+\alpha)$; α is the clutter to signal ratio
- **SNR theory derived and shown to agree with MC measurements**
  - Best SNR not necessarily at best efficiency
  - Linear SNR theory valid when $m_s < 1/(1+\alpha)$
  - Non-linear device SNR theory based on binomial statistics agrees with measurements
  - When SNR > Linear Theory limit the device is saturated and targete reflectance information is lost!
- **Detection Statistics Derived and compared to Coherent Detection and Linear Photon Counting DD**
  - Linear PC is better than GAPD due to PrA